Draft version February 2, 2008
Preprint typeset using LaTeX style emulateapj v. 11/12/01

DETECTION OF GRAVITATIONAL WAVES FROM THE COALESCENCE OF POPULATION-III REMNANTS WITH ADVANCED LIGO

J. Stuart B. Wyithe[1] and Abraham Loeb[2]
swyithe@isis.ph.unimelb.edu.au; aloeb@cfa.harvard.edu

Draft version February 2, 2008

ABSTRACT

The comoving mass density of massive black hole (MBH) remnants from pre-galactic star formation could have been similar in magnitude to the mass-density of supermassive black holes (SMBHs) in the present-day universe. We show that the fraction of MBHs that coalesce during the assembly of SMBHs can be extracted from the rate of ring-down gravitational waves that are detectable by Advanced LIGO. Based on the SMBH formation history inferred from the evolution of the quasar luminosity function, we show that an observed event rate of 1 yr^{-1} will constrain the SMBH mass fraction that was contributed by MBHs coalescence down to a level of $\sim 10^{-6}$ for $20M_\odot$ MBH remnants (or $\sim 10^{-4}$ for $260M_\odot$ remnants).

Subject headings: black hole physics - cosmology: theory - galaxies: formation

1. INTRODUCTION

The first episode of star formation (Pop-III) occurred in metal-free gas that cooled to low temperatures through collisional excitation of molecular hydrogen, H_2 (Bromm & Larson 2003). The first stars could have formed inside mini-halos of masses $M \gtrsim 10^5 M_\odot$ at redshifts $z \gtrsim 20$ (Barkana & Loeb 2001). Numerical simulations indicate that the first stars were probably more massive than the sun by two orders of magnitude (Bromm, Coppi & Larson 1999; Abel, Bryan & Norman 2000). Pop-III stars with a mass $M_\star \gtrsim 260 M_\odot$ or $25 \lesssim (M_\star/M_\odot) \lesssim 140$ end their life by forming a MBH remnant (Heger & Woosley 2002; Heger et al. 2003).

The formation of Pop-III stars must have been self-regulating, as the UV radiation emitted by these stars dissociated H_2 throughout the universe (Haiman, Rees & Loeb 1997) and boiled (via photo-ionization heating) gas out of shallow potential wells with a virial temperature $\lesssim 10^4$ K (Barkana & Loeb 1999), whereas supernovae explosions expelled gas from even deeper potential wells (Yoshida et al. 2003). Moreover, Pop-III supernovae enriched star-forming regions with metals above the critical value of $\sim 0.1\%$ of the solar metallicity, allowing fragmentation of the gas into lower-mass stars (Bromm & Loeb 2003). Even with this self-regulation included, Madau & Rees (2001) estimate that the co-moving mass density in MBHs at $z \sim 20$ may have been comparable to the observed mass density of supermassive black holes (SMBHs) in the present-day universe. Thus, in principle, the coalescence of MBH remnants could have made a significant contribution to the SMBH mass budget.

More recent semi-analytic calculations of the merger history of SMBHs (Islam, Taylor & Silk 2002) find that although the mass density in Pop-III MBH remnants may be sufficient to provide the present day density of SMBHs, most of the MBHs end up in satellites and populate galaxy halos. Nevertheless, hierarchical merging inevitably leads to the formation of early dwarf galaxies, which could sink via dynamical friction to make dense clusters of MBHs at the centers of massive galaxies that are assembled later (Madau & Rees 2001). These central MBH clusters provide sites for MBH coalescence and could produce seeds for SMBH growth (analogous processes take place within the segregated cores of globular star clusters; see Portegies-Zwart & McMillan 2000).

The rise of the SMBH population is traced by the evolution of the quasar luminosity function (e.g. Yu & Tremaine 2002). While the gas accretion history of the quasar population accounts for most of the locally observed SMBH mass, coalesced MBHs may also contribute some small fraction, $f_{\rm MBH}$, of that mass. For a given value of $f_{\rm MBH}$, the coalescence rate of MBHs corresponding to the buildup of mass in SMBHs may then be calculated based on the observed quasar history. In this *Letter* we show that detection of gravitational waves from MBH coalescences by *Advanced LIGO*[3] (LIGO-II) will constrain the fraction of the SMBH mass that was assembled through coalescence of MBHs and provide a direct probe of the early population of very massive stars.

2. RING-DOWN GRAVITATIONAL WAVES FROM MBHS AND DETECTION BY LIGO-II

First, we consider the expected gravitational radiation signal from merging MBHs at high redshifts. The coalescence process of two black-holes can be separated into three phases: an inspiral phase, a coalescence phase, and finally a ringing phase of the merged black-hole product (Flanagan & Hughes 1998) as it settles to a Kerr metric. Of these, the inspiral and ringing phases generate well understood wave-forms. Inspiraling MBHs at high redshift will be most luminous at frequencies near 1 Hz, just below the sensitivity band for LIGO-II. However, following a merger the MBHs would ring at higher frequencies that are within the LIGO-II sensitivity band.

To compute the gravitational wave strain for black-hole ringing we follow the formalism outlined by Hughes (2002).

[1] University of Melbourne, Parkville, Victoria, Australia
[2] Harvard-Smithsonian Center for Astrophysics, 60 Garden St., Cambridge, MA 02138
[3] http://www.ligo.caltech.edu/advLIGO/

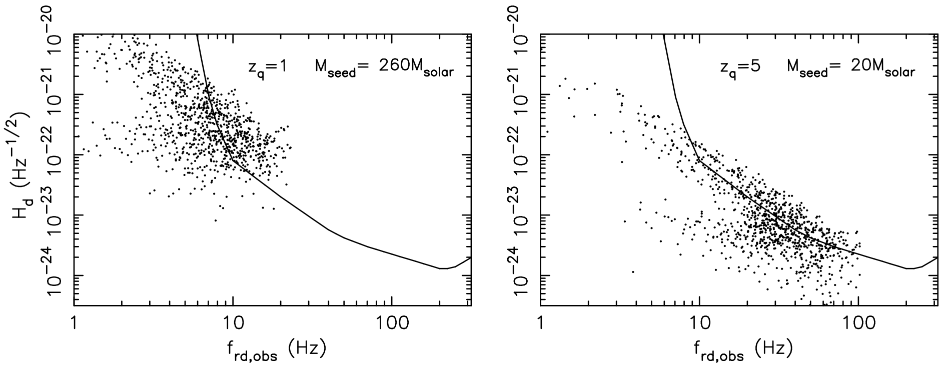

FIG. 1.— Comparison between the ring-down strain signals $H_{\rm d}$ and the LIGO-II sensitivity curve as a function of observed frequency, $f_{\rm rd,obs}$. The points correspond to hierarchical coalescence of 10^3 MBH seeds to a single MBH at a redshift $z_{\rm q}$. Two cases are shown, having MBH seeds of $20M_\odot$ and $260M_\odot$, coalescing by $z_{\rm q}=5$ and $z_{\rm q}=1$, respectively. Each coalescence event involves a different randomly-chosen combination of the parameters θ, ϕ, ψ, λ and $\hat{L}\cdot\hat{n}$. The calculations assume $\epsilon_{\rm rd}=0.03$ and $a=1.0$.

The dimensionless strains for the two polarizations of the gravitational waveform during the ring down may be written as

$$h_{+,-}(t)=\mathcal{A}_{+,-}\exp\left(-\pi f_{\rm rd,obs}t/Q\right)\cos\left(2\pi f_{\rm rd,obs}t\right), \quad (1)$$

where $Q\equiv\pi f_{\rm rd,obs}\tau_{\rm obs}$ is the quality factor, $\tau_{\rm obs}$ is the observed decay time and t is the observed time following the merger. The observed frequency, $f_{\rm rd,obs}$, equals the intrinsic frequency, $f_{\rm rd}$, divided by $(1+z)$. Assuming that the ratio of the strain amplitudes for the two polarizations follows that of the inspiral phase,

$$\mathcal{A}_+=\mathcal{A}_{\rm ring}\left[1+(\hat{L}\cdot\hat{n})^2\right]\ ;\ \mathcal{A}_\times=-2\mathcal{A}_{\rm ring}(\hat{L}\cdot\hat{n}), \quad (2)$$

where $\hat{L}\cdot\hat{n}$ is the dot product of the unit vectors pointing along the binary orbital angular momentum and the line of sight to the source. The overall strain amplitude $\mathcal{A}_{\rm rd}$ is found by requiring that the binary radiates a fraction $\epsilon_{\rm rd}$ of its energy (e.g. Fryer, Holz & Hughes 2002). If a black hole of mass $\Delta M_{\rm bh}$ merges with a black hole of mass $M_{\rm bh}$

$$\mathcal{A}_{\rm rd}=10^{-23}\left(\frac{D}{10\ {\rm Gpc}}\right)^{-1}\sqrt{\frac{20\epsilon_{\rm rd}}{4\pi f_{\rm rd}Q}\left(\frac{M_{\rm bh}+\Delta M_{\rm bh}}{M_\odot}\right)} \quad (3)$$

Note that in this expression $D=D_{\rm L}/(1+z)$ is the co-moving distance, where $D_{\rm L}$ is the luminosity distance (e.g. Hogg 1999). The energy flux is proportional to $(f_{\rm rd,obs})^2h^2_{\rm ring}\propto\left[(f_{\rm rd})^2(1+z)^{-2}\right]D^{-2}\propto D_{\rm L}^{-2}$.

Distortions of the Kerr metric during the ringing phase may be decomposed into spheroidal modes with spherical harmonic like indicies l and m (Fryer, Holtz & Hughes 2002). The quadrupole ($l=2$) moments are expected to dominate ringdown spectrum. Binary coalescence excites a bar-like deformation of the event horizon (Hughes 2002). As a result, the $m=2$ mode dominates over the $m=0$ mode when the ringdown forms the final stage of a binary merger. Fits to numerical simulation show that for $l,m=2$ (Leaver 1985; Echeverria 1989; Fryer, Holtz & Hughes 2002)

$$f_{\rm rd}\sim\frac{10^{5.3}{\rm Hz}}{2\pi}\left(\frac{M_{\rm bh}+\Delta M_{\rm bh}}{M_\odot}\right)^{-1}\left[1-0.63(1-a)^{3/10}\right], \quad (4)$$

and

$$Q=2(1-a)^{-9/20}, \quad (5)$$

where a is the dimensionless black-hole spin, taken to have representative values of 1, 0.5 or 0 in the numerical results presented later. Hughes & Blandford (2002) have found that equal mass mergers may have rapidly rotating remnants ($a\sim 1$), while mass ratios of 0.5 lead to typical spins of $a\sim 0.5$. We note that in addition to setting the oscillation frequency, the spin governs the value of Q and hence the number of oscillations. Ringdown waveforms with more oscillations will be easier to detect.

The observed strain may be written as

$$H(t)=F_+(\theta,\phi,\psi)h_+(t)+F_\times(\theta,\phi,\psi)h_\times(t), \quad (6)$$

where $F_+(\theta,\phi,\psi)$ and $F_+(\theta,\phi,\psi)$ are the detector response functions for a source with a sky position (θ,ϕ) and polarization axes rotated at an angle ψ (e.g. Thorne 1987). Since the ring-down waves lie in a narrow frequency range, the signal-to-noise ratio ρ for a detection utilizing matched-filtering techniques may be evaluated as (Hughes 2002)

$$\rho^2=\frac{2\int_0^\infty dtH(t)^2}{S_{\rm h}(f_{\rm rd,obs})}, \quad (7)$$

where $S_{\rm h}(f_{\rm rd,obs})$ is the spectral density of detector noise (in ${\rm Hz}^{-1}$).

3. MBH MERGERS AND THE EVOLUTION OF THE SMBH POPULATION

In the redshift interval between $z_{\rm q}$ and $z_{\rm q}+dz_{\rm q}$, the accretion due to quasars implies that the co-moving mass density in SMBHs is

$$d\rho_{\rm bh}(z_{\rm q})=dz_{\rm q}\int_0^\infty dL_{\rm B}\frac{L_{\rm bol}}{\epsilon c^2}\Psi(L_{\rm B},z_{\rm q})\frac{dt}{dz_{\rm q}}, \quad (8)$$

where $L_{\rm bol}$ is the bolometric luminosity of a quasar with a B-band luminosity $L_{\rm B}$, ϵ is the conversion efficiency of accreted mass to electromagnetic radiation (we adopt $\epsilon=0.1$; Yu & Tremaine 2001), and $\Psi(L_{\rm B},z_{\rm q})$ is the co-moving density of quasars per unit B-band luminosity at $z_{\rm q}$, for which we adopt the parametric form derived from the 2dF survey (Boyle et al. 2000). By integrating equation (8) over redshift, one may obtain the SMBH mass-density accreted by a redshift $z_{\rm q}$. The local ($z_{\rm q}=0$) value for the co-moving mass density in SMBHs accreted during the quasar activity is $\rho_{\rm bh}\approx 2.7\times10^5M_\odot{\rm Mpc}^{-3}$. This number is similar to estimates of the local SMBH mass

density obtained from the inventory of quiescent galactic SMBHs (e.g. Yu & Tremaine 2002; Aller & Richstone 2002).

If some fraction of the SMBH mass is assembled through coalescence during the hierarchical build-up of galaxies, then it is natural to expect MBH remnants to contribute to the SMBH mass budget as they are more massive than normal stellar remnants and should sink to the centers of galaxies by dynamical friction (Madau & Rees 2001). The initial coalescence of the MBHs would be detectable by Advanced LIGO, and the observed event rate (or lack thereof) could shed light on the possibility that MBH coalescence events produced the early seeds of SMBHs. Suppose that a fraction $f_{\rm MBH}$ of the mass in SMBHs at any redshift was added through coalescence of MBHs, with the remainder added through gas accretion. Since the comoving mass density of accreted gas mass for quasars is comparable to the final SMBH mass density in the local universe, we expect $f_{\rm MBH} \ll 1$ independent of other considerations.

While the accreted mass is being added at redshift $z_{\rm q}$, the coalescence of the MBHs incorporated into the SMBH at $z_{\rm q}$ would have stretched over most of the age of the universe $t_{\rm age}(z_{\rm q})$, corresponding to the assembly time of the SMBH. The MBHs are brought together through hierarchical merging of the mini-halos that form the building blocks for the massive dark matter halo hosting any particular SMBH. There are several issues to consider, including the number and clustering of MBHs inside mini halos, the merger rate of mini-halos into proto-galaxies and more massive hosts, and the subsequent dynamical behavior of the MBHs within the resulting gaseous and stellar environments. Rather than model the coalescence history using a dark-matter halo merger tree and the poorly understood dynamics of the MBH population, we instead assume that the MBHs undergo random hierarchical coalescences with the MBH mergers distributed linearly in time. At any point in the coalescence history, two MBHs that coalesce are chosen at random from the MBH population. The next merger is then drawn from the revised population which contains one fewer MBH in total, and one additional MBH of a larger mass, and so on. There are a total of $N-1$ coalescence events leading from the N initial seed MBHs to the single final MBH of mass $NM_{\rm MBH}$. One third of these coalescence events are between MBHs of the initial seed mass.

Figure 1 shows the signal amplitude $H_{\rm d} \equiv \sqrt{2\int_0^\infty dt H(t)^2}$ for the hierarchical coalescence of 10^3 MBH seeds to a single MBH at a redshift $z_{\rm q}$. Two cases are illustrated, having MBH seeds of $20M_\odot$ and $260M_\odot$ coalescing by $z_{\rm q}=5$ and $z_{\rm q}=1$, respectively. Each individual coalescence event was combined with a different randomly chosen combination of the parameters θ, ϕ, ψ, λ and $\hat{L}\cdot\hat{n}$. The calculations assume $\epsilon_{\rm rd}\sim 0.03$ (Hughes 2002), and $a=1.0$. The values of $H_{\rm d}$ in Figure 1 lie along two branches. The lower branch originates from the early stage of the merger tree, where the seed MBHs provide most of the coalescence events. As the typical mass of the coalescing MBHs grows at lower redshifts, the signal amplitude increases while its frequency decreases. The figure also shows $\sqrt{S_{\rm h}(f_{\rm rd,obs})}$, the planned sensitivity curve for the two LIGO-II 4km interferometers (Gustafson, Shoemaker, Strain & Weiss 1999). The signal-to-noise ratio ρ is the ratio between the points and the sensitivity curve [see equation (7)]. From an ensemble of such detection histories, we may therefore compute the fraction of mergers $f_{\rm detect}(z_{\rm q}, z_{\rm coal})$ that are detectable by LIGO-II as a function of the quasar redshift, $z_{\rm q}$, and the coalescence redshift, $z_{\rm coal}$.

The rate at which coalesced MBH mass is added to the SMBH population is given by

$$\frac{d\rho_{\rm MBH}}{dz_{\rm q}} = \frac{f_{\rm MBH}}{1-f_{\rm MBH}}\frac{d\rho_{\rm bh}}{dz_{\rm q}}, \tag{9}$$

with the remainder added via gas accretion during the active quasar phase. The number density increment of consumed MBHs can be derived from the mass density increment, $dn_{\rm MBH}(z_{\rm q}) = d\rho_{\rm MBH}(z_{\rm q})/M_{\rm MBH}$.

The redshift distribution of the rate of detectable coalescences (in the observer frame) is

$$\frac{d^2N_{\rm det}}{dt dz_{\rm coal}} = \int_\infty^0 dz_{\rm q}\left[dn_{\rm MBH}4\pi\frac{dV}{d\Omega dz_{\rm q}}\frac{f_{\rm detect}(z_{\rm q},z_{\rm coal})}{[t_{\rm age}(z_{\rm q})]^2}\right.$$
$$\left.\frac{dt}{dz_{\rm coal}}\left(\frac{1}{1+z_{\rm coal}}\right)\Theta(z_{\rm coal}-z_{\rm q})\right], \tag{10}$$

where the factor $1/(1+z_{\rm coal})$ is due to cosmological time dilation, the derivative $dt/dz_{\rm coal}$ enforces a linear distribution of coalescence events in time, $(dV/d\Omega dz_{\rm q})$ is the co-moving volume per unit solid angle per unit redshift, and $\Theta(x)$ is the Heaviside step function.

Figure 2 shows $d^2N_{\rm det}/dtd\ln(1+z_{\rm coal})$ in units of $10^5 f_{\rm MBH}\ {\rm yr}^{-1}$. The plotted curves correspond to a signal-to-noise ratio of 5, assuming a MBH initial mass of $20M_\odot$ (left panels) and $260M_\odot$ (right panels). The calculations were performed for values of $a=1.0$ and $\epsilon_{\rm rd}=0.01$, 0.03 and 0.10. A fraction as small as $f_{\rm MBH}\sim 10^{-5}$ yields ~ 0.1–10 detectable coalescence events at redshifts $2\lesssim z_{\rm q}\lesssim 5$ (where the quasar population is substantial and the observed frequency does not redshift out of the Advanced LIGO band above 10 Hz). For comparison, the light lines show the redshift distribution of the entire coalescence rate ($f_{\rm detect}=1$) in the observer frame, $d^2N_{\rm mrg}/dtd\ln(1+z_{\rm coal})$. We find that Advanced LIGO will be able to detect most of the low redshift coalescences, while the detectable fraction declines at early cosmic times due to the redshift in frequency.

Values of the total event rate $(dN_{\rm det}/dt)$ are presented in Table 1 in units of $10^5 f_{\rm MBH}\ {\rm yr}^{-1}$. In addition to the examples plotted in Figure 2, the table lists results for BH spins of $a=0.0$ and $a=0.5$. The rates are lower for smaller MBH spins and for larger MBH masses, due to the lower emission frequency and smaller numbers of MBHs.

4. DISCUSSION

We have estimated the number of seed MBHs that must have coalesced at high redshifts in order to contribute a fraction $f_{\rm MBH}$ of the mass density in quasar SMBHs. We then found the corresponding detection rate of ring-down gravitational waves for the Advanced LIGO interferometers. If the initial mass of MBH remnants was $20M_\odot$, then 1 event per year would imply a coalesced mass fraction of $f_{\rm MBH}\sim 10^{-6}$. On the other hand, if the MBH seed mass was $260M_\odot$, then 1 event per year would imply a coalesced mass fraction of $f_{\rm MBH}\sim 10^{-4}$. Advanced LIGO will be primarily sensitive to MBH coalescence at $z\sim 2-3$, while

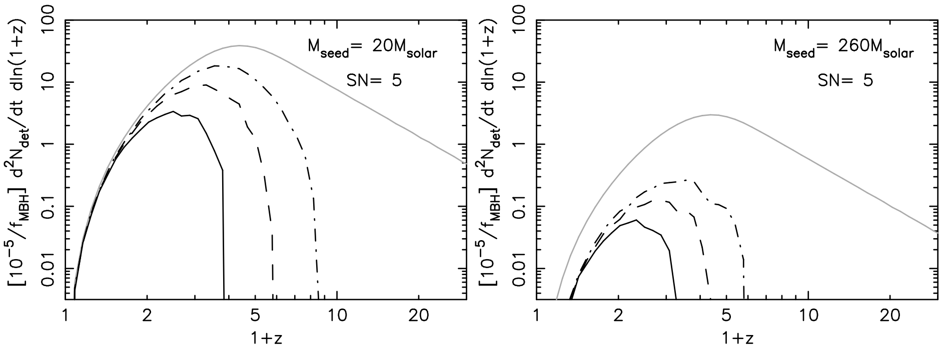

FIG. 2.— The rate of detectable mergers per $\ln(1+z)$ in units of $10^5 f_{\rm MBH}$ yr^{-1}. Curves are shown for a signal-to-noise ratio of 5, and initial MBH mass of $20M_\odot$ (left panels) and $260M_\odot$ (right panels). Each panel shows three curves for $\epsilon_{\rm rd} = 0.01$ (solid), 0.03 (dashed) and 0.10 (dot-dashed). The calculations assumed $a = 1.0$. For comparison, we also show the total number of mergers per year in the observer frame (light lines).

TABLE 1
DETECTION RATE OF COALESCENCE EVENTS IN UNITS OF $10^5 f_{\rm MBH}$YR^{-1}. THE THIRD COLUMN SHOWS THE MERGER RATE ASSUMING THAT ALL EVENTS CAN BE DETECTED.

			$[10^{-5}$ yr$/f_{\rm MBH}] \times dN_{\rm det}/dt$		
$M_{\rm seed}$	a	$[10^{-5}/f_{\rm MBH}] \times dN_{\rm mrg}/dt$	$\epsilon_{\rm rd} = 0.01$	$\epsilon_{\rm rd} = 0.03$	$\epsilon_{\rm rd} = 0.10$
$20M_\odot$	1.0	35.6	1.9	5.7	13.3
$20M_\odot$	0.5	35.6	1.0	3.4	9.8
$20M_\odot$	0.0	35.6	0.67	2.3	7.5
$260M_\odot$	1.0	2.7	0.02	0.07	0.15
$260M_\odot$	0.5	2.7	0.003	0.01	0.03
$260M_\odot$	0.0	2.7	0.001	0.003	0.007

our calculation implies that most of the MBHs for which coalescence is possible may have coalesced earlier. However, values of $f_{\rm MBH} \sim 10^{-4} - 10^{-6}$ correspond to only a few events per typical galactic nucleus per Hubble time, a very small coalescence rate that is comparable to the rate by which massive galaxies merge.

It is possible that not all MBH coalescences contribute to the assembly of the SMBH mass. In particular, MBHs that are born into tight binaries might coalesce, even if they do not migrate into dense stellar systems or MBH clusters. If MBH binaries did coalesce without adding mass to the assembled SMBHs, then the observed event rate would only provide an upper limit on $f_{\rm MBH}$. MBHs that coalesced directly with SMBHs without first undergoing coalescence with other MBHs would be invisible to Advanced LIGO, but might be detected by the planned space mission *LISA*[4] (Wyithe & Loeb 2003).

We thank Scott Hughes and Andrew Melatos for helpful comments. This work was supported in part by NASA grant NAG 5-13292, and by NSF grants AST-0071019, AST-0204514 for AL.

REFERENCES

Abel, T., Bryan, G. L., & Norman, M. L. 2000, ApJ, 540, 39
Aller, M. C. & Richstone, D. 2002, AJ, 124, 3035
Boyle, B. J., Shanks, T., Croom, S. M., Smith, R. J., Miller, L., Loaring, N., & Heymans, C. 2000, MNRAS, 317, 1014
Barkana, R., & Loeb, A. 2001, Phys. Rep., 349, 125
Barkana, R., & Loeb, A. 1999, ApJ, 523, 54
Bromm, V., & Larson, R. B. 2003, ARA&A, in press; astro-ph/0311019
Bromm, V., Coppi, P. S., & Larson, R. B. 1999, ApJ, 527, L5
Bromm, V., & Loeb, A. 2003, Nature, 425, 812
Echeverria, F. 1989, Phys. Rev. D, 40, 3194
Flanagan, É. É. & Hughes, S. A. 1998, Phys. Rev. D, 57, 4535
Fryer, C. L., Holz, D. E., & Hughes, S. A. 2002, ApJ, 565, 430
Gustafson, E., Shoemaker, D., Strain, K., & Weiss, R. 1999, LSC White Paper on Detector Research and Development (LIGO project document T990080-00-D)
Haiman, Z., Rees, M., & Loeb, A. 1997, ApJ, 476, 458 (erratum 484, 985)
Heger, A. & Woosley, S. E. 2002, ApJ, 567, 532
Heger, A., Fryer, C. L., Woosley, S. E., Langer, N., & Hartmann, D. H. 2003, ApJ, 591, 288
Hogg, D., 1999, astro-ph/9905116
Hughes, S. A. 2002, MNRAS, 331, 805
Hughes, S. A. & Blandford, R. D. 2003, ApJ, 585, L101
Islam, R. R., Taylor, J. E., & Silk, J. 2003, MNRAS, 340, 647
Leaver, E. W. 1985, Royal Society of London Proceedings Series A, 402, 285
Madau, P. & Rees, M. J. 2001, ApJ, 551, L27
Portegies Zwart, S. F. & McMillan, S. L. W. 2000, ApJ, 528, L17
Sigurdsson, S., Hernquist, L., 1993, Nature, 364, 423
Thorne, K.S., 1987, in Hawking, S. & Israel, W., eds, 300 Years of Gravitation. Cambridge Univ. Press, Cambridge, p. 330
Wyithe, J. S. B. & Loeb, A. 2003, ApJ, 590, 691
Yoshida, N., Bromm, V., & Hernquist, L. 2003, ApJ, submitted; astro-ph/0310443
Yu, Q. & Tremaine, S. 2002, MNRAS, 335, 965

[4] http://lisa.jpl.nasa.gov/